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Cosmological Implications of the Supergravity Tracking Potential

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Abstract. It is demonstrated that any realistic model of quintessence should be based on Supergravity since, when the scalar field is on tracks today, $Q \approx m_{\rm Pl}$. This improves the agreement between theoretical predictions and the current observations. In particular, a generic property is that ratio $\omega_{\rm Q} \equiv p_{\rm Q}/\rho_{\rm Q}$ is pushed towards -1. A string-inspired model is proposed where the potential is given by $V(Q) = \frac{\Lambda^{4+\alpha}}{Q^\alpha} e^{\frac{\kappa}{2}Q^2}$. The model predicts $\omega_{\rm Q} \approx -0.82$, a value less that one sigma from the current likehood value.

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1 Introduction

There are now several hints indicating that our Universe could presently undergo a phase of accelerated expansion. These hints consist in a set of recent observations including particulary (but not only) measurements of the Hubble law using type Ia supernovae [1]. All these observations seem to point towards the same conclusion: a fluid with a negative pressure to energy density ratio contributes for 70% of the matter content of the Universe and therefore represents the dominant component of this matter content.

If this conclusion is confirmed, this immediately raises the question of the physical nature and origin of this fluid. At first sight, a natural candidate is the cosmological constant. A cosmological constant is equivalent to a fluid with an equation of state given by $p = -\rho$. This seems in agreement with the recent analysis of the data which indicate that $-1 < p/\rho < -0.8$ or $-1 < p/\rho < -0.6$ according to Refs. [2] and [3]. However, $\Omega_\Lambda \approx 0.7$ corresponds to an energy scale of $\approx 5.7 \times 10^{-47}\mathrm{GeV}^4$ which is very far from the natural scales of High Energy Physics. Therefore, even if there is presently no contradiction with observations, it seems that this hypothesis runs into theoretical problems.

Another explanation was recently put forward in Refs. [4]. It consists in assuming that the unknown fluid is a scalar field named quintessence. Then, the following

questions need to be answered. Firstly, why is it so that this fluid is dominating today? This is the coincidence problem. Secondly, in order to explain $\Omega_{\rm Q} \approx 0.7$, is it necessary to fine tune a free parameter of the theory to a value very far from the natural scales of Particle Physics? In that case, nothing would have been gained in comparison with the cosmological constant case. This is the fine tuning problem. Thirdly, is it possible to find values of the pressure to energy density ratio compatible with recent findings? This is the equation of state problem. Fourthly, can realistic quintessence models be implemented in the realm of High Energy Physics? This is the model building problem.

It has been argued in Refs. [4] that these four problems can be partially tackled if the potential is given by:

$$V(Q) = \frac{\Lambda^{4+\alpha}}{Q^\alpha}, \tag{1}$$

where Λ and $\alpha > 0$ are free parameters. The coincidence problem is solved because the equation of motion of the scalar field possesses a tracking solution. Just after reheating, at a redshift of $z \approx 10^{28}$, the allowed initial conditions for the energy density of the quintessence field are $10^{-37}\mathrm{GeV}^4 < \rho_{\rm Q} < 10^{61}\mathrm{GeV}^4$ (i.e. 100 orders of magnitude!). For any initial value in this range, the field is led to the same solution. The fine tuning problem is partially solved. For $\alpha = 11$, for example, $\Lambda \approx 10^{10}\mathrm{GeV}$ in order to have $\Omega_{\rm Q} \approx 0.7$. Therefore the natural scale of the problem is now comparable to the natural scales of Particle Physics. However, it is still necessary to adjust this free parameter. The main problem for the models based on the potential (1) could come from the equation of state. It has been show that $\omega_{\rm Q} \equiv p_{\rm Q}/\rho_{\rm Q}$ cannot be less than -0.7. This seems in disagreement with observations if one believes in the estimates of Ref. [2]. However, according to Ref. [3], there is still an open windows for these models. Finally, the question of the model building have been adressed, in particular, in Ref. [5] where a model based on global supersymetry (SUSY) has been proposed. In this model, the Kähler potential is flat, $K(Q,Q^*) = QQ^*$, and the superpotential is given by $W(Q) = \Lambda^{3+a}/Q^a$. This leads to an inverse power law scalar potential.

The aim of this article is to show that any realistic model of quintessence must be based on Supergravity (SUGRA). We proove that, generically, taking into account SUGRA improves the agreements with observations, especially with regards to the equation of state. In order to illustrate these general properties, we exhibit a specific model where concrete calculations can be performed and we propose a new potential for the quintessence field, the supergravity tracking potential.

2 Taking into account SUGRA

The questions evoked in the previous section can be adressed if the field is on tracks. According to Ref. [4], this means that it should satisfy the equation:

$$\frac{\mathrm{d}^2 V(Q)}{\mathrm{d}Q^2} = \frac{9}{2}\frac{\alpha+1}{\alpha}(1-\omega_{\rm Q}^2)H^2, \tag{2}$$

where H is the Hubble constant. This equation implies $Q \approx m_{\rm Pl}$ now. Since SUGRA corrections are of the order $Q/m_{\rm Pl}$, they are crucial for quintessence and any realistic model should be based on SUGRA.

The SUGRA ($N = 1$, $D = 4$) scalar potential is given by $V = \kappa^{-2}e^G(G^iG_i-3)+V_{\rm D}$, where $\kappa \equiv 8\pi/m_{\rm Pl}^2$, $G \equiv \kappa K + \ln(\kappa^3|W|^2)$ and $V_{\rm D} \geq 0$ is a term coming from the gauge sector. Then, we can draw general conclusions. Firstly, the Kähler and super potentials should be chosen such that negative contributions do not dominate the scalar potential. This is not the case if the SUGRA corrections to the model proposed in Ref. [5] are taken into account. Secondly, we see that for a typical polynomial Kähler potential K, the term e^G is unimportant throughout almost all the cosmic evolution since $Q \ll m_{\rm Pl}$. This means that the term G^iG_i should be responsible for the tracking properties. Thirdly, the term e^G should dominate now since we have $Q \approx m_{\rm Pl}$. Since the exponential is a rapidly growing function, this implies that the potential energy should largely dominate the kinetic energy today. In other words, this automatically pushes the ratio $\omega_{\rm Q}$ towards -1 which is precisely needed in order not to be in conflict with observations. These properties are generic, i.e. they do not depend on the details of the model, even if of course it is certainly possible to find very specific cases where they are not true[a].

In Refs. [6, 7], string-inspired Kähler and superpotentials were proposed. They lead to the SUGRA tracking potential given by:

$$V(Q) = \frac{\Lambda^{4+\alpha}}{Q^\alpha} e^{\frac{\kappa}{2}Q^2}, \quad \alpha > 11. \tag{3}$$

In the next section, we investigate in more details the cosmological implications of this potential.

3 Cosmological implications

In order to study the properties of the SUGRA tracking solutions, we have numerically integrated the full Einstein equations. We have checked that the coincidence problem is still solved in the SUGRA case, see Refs. [6, 7]. The evolution of $\omega_{\rm Q}$ is displayed in Fig. (1). The influence of the exponential term at the end of the evolution, for small redshifts, is clearly visible. For $\alpha = 11$, the value of the equation of state for the potential given by Eq. (1) is $\omega_{\rm Q} \approx -0.29$, a value ruled out by observations. For the SUGRA tracking potential, we find:

$$\omega_{\rm Q} \approx -0.82, \tag{4}$$

for $\Omega_{\rm Q} \approx 0.7$. According to Refs. [2], this is less than one sigma from the likehood value. For $\Omega_{\rm Q}$ between 0 and 1, $\omega_{\rm Q}$ changes between -0.22 and -0.995. If $\Omega_{\rm Q} \approx 0.75$ then $\omega_{\rm Q} \approx -0.86$. Furthermore, these numbers do not require any fine tuning of the free parameter α. This is because the value is mainly determined by the exponential factor which is α independent. This property is illustrated in Fig. (2).

In conclusion, we would like to emphasize that taking into account SUGRA is mandatory if one wishes to construct a realistic model of quintessence. This is because

[a] In particular a logarithmic Kähler potential for string moduli has been ruled out [7]

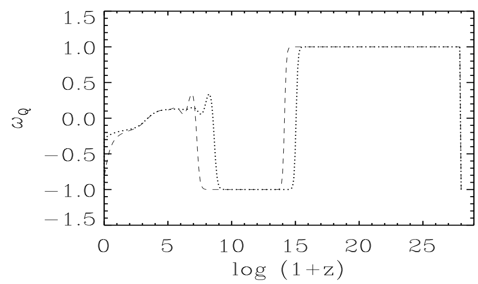

Fig. 1 The dotted line represents the evolution of $\omega_{\rm Q}$ for the potential given in Eq. (1) with $\alpha = 11$ whereas the dashed line represents the evolution of $\omega_{\rm Q}$ for the SUGRA tracking potential, Eq. (3), with the same value of α.

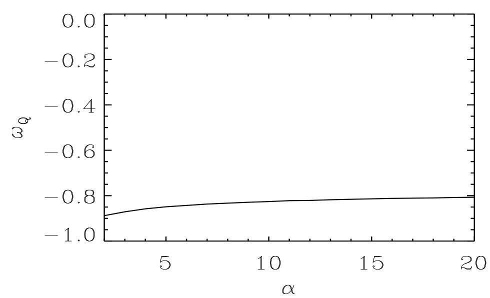

Fig. 2 $\omega_{\rm Q} - \alpha$ relation for the SUGRA tracking potential

when the field is on tracks, $Q \approx m_{\rm Pl}$. Then, a natural consequence is that theoretical predictions fit better the current data. In particular, the model presented here predicts $\omega_{\rm Q} \approx -0.82$ which lies into the one sigma error interval.

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